THE UNIVERSITY OF BRITISH COLUMBIA
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 466 – AUTOMATIC CONTROL

Final Examination

First Term 2006/2007

07 December 2006

- **Duration: Three Hours**
- **Open Book/Notes**
- **Electronic equipment, including calculators, is not allowed**
- **Fully answer all three problems for full credit**
- **Clearly state all the assumptions made and give all steps of the derivations**
- **If you use new notations, you must clearly define them**
- **This exam paper contains four pages including the cover sheet.**

Problem 1

(a) List three parameters each, which can be used to specify the performance of a control system with respect to:

(i) Speed of response

(ii) Relative stability

Define these parameters.

(5 Points)

(b) Consider the feedback control system given by the block diagram in Figure 1. The forward transfer function is denoted by $G(s)$, which represents the plant and part of the controller. The feedback transfer function is given by $H(s) = bs + 1$.

(i) As a hardware component of the system, what does this feedback transfer function represent?

(5 points)

The following facts are known about the control system:

1. It is a third order system but behaves almost like a second order damped oscillator (with no zeros)
2. It is a Type 2 system
3. Its 2% settling time is 4/3 seconds, for a step input.
4. Its peak time is $\frac{\pi}{4}$ seconds, for a step input.

(ii) Completely determine the third order forward transfer function $G(s)$ (i.e., the numerical values of all its parameters) and the parameter b in the feedback transfer function.

(15 points)

(iii) Estimate the damping ratio and the percentage overshoot of the closed-loop system.

(5 points)

(iv) If a unit ramp input is applied to the system, what is the resulting steady state error?

(5 points)

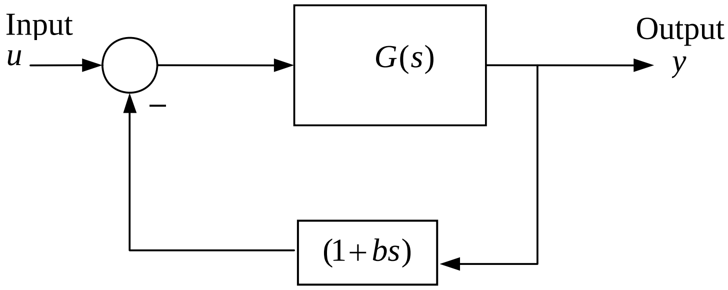

Figure 1 A feedback control system.

Problem 2

(a) The poles of a system are given in the following five examples. In each case state giving reasons whether the system is stable, unstable, or marginally stable.

(i) -2, $-4 \pm j5$, -3
(ii) -2, $-4 \pm j5$, +3
(iii) -2, $-4 \pm j5$, $\pm j3$
(iv) -2, $+4 \pm j5$, -3
(v) $-4 \pm j5$, 0, 0

(5 points)

(b) A system has the characteristic equation

$$s^3 + 12s^2 + 61s + 150 = 0$$

(i) Using Routh-Hurwitz criterion determine whether the system is stable, unstable, or marginally stable.

(10 points)

(ii) Now move all the poles of the given system to the right of the s-plane by the real value 3. (i.e., add 3 to every pole of the original system). Using Routh-Hurwitz criterion determine whether this modified system is stable, unstable, or marginally stable. Justify your answer.

(5 points)

(iii) Using the result in Part (ii) and without directly solving the characteristic equation determine all three poles of the original system.

(10 points)

Note: Give all the details of obtaining your results. You should answer this question **<u>without</u>** directly solving a 3rd order characteristic equation.

Problem 3

A system is shown in Figure 3. It was found to have the following properties:

1. The system transfer function $G(s)$ has two zeros and three poles.
2. The product of the three poles is -4.
3. When the system was excited with a sinusoidal input *u* (as shown in Figure 3) at frequency $\omega = 4,$ the output *y* at steady state was found to be zero (i.e., no response).
4. When the system was excited with a sinusoidal input *u* (as shown in Figure 3) at frequency $\omega = 2,$ the output *y* at steady state was found to have a phase lag of 180º with respect to the input (i.e., the response was in the opposite direction to the input).
5. When the system was excited with a sinusoidal input *u* (as shown in Figure 3) at frequency $\omega = \sqrt{2},$ the output *y* at steady state was found to have a phase lag of 90º with respect to the input.
6. The DC gain of the system (i.e., the magnitude of the frequency transfer function at zero frequency) is 8.

Determine the complete transfer function $G(s)$ of the system (i.e., the numerical values of the five parameters in $G(s)$).

(35 Points)

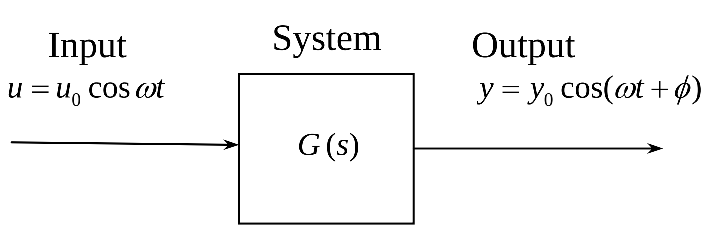

Figure 3 An open-loop system.